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LETTER TO THE EDITOR

Criterion for unbroken supersymmetry at finite temperature

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Abstract. The conditions for unbroken supersymmetry at finite temperature are studied. It is shown that, at finite temperature, vanishing of thermal averages of all the auxiliary fields alone is not sufficient for supersymmetry to remain unbroken.

In recent years supersymmetry at finite temperature (T) has been studied by many authors. They tried to establish the criterion for unbroken supersymmetry (SUSY) at non-zero temperature. It seems there is still some controversy regarding the behaviour of SUSY at finite T. Some authors (Das and Kaku 1978, Girardello *et al* 1981, Boyanovsky 1984) feel that SUSY is automatically broken as soon as thermal effects are taken into account. However, there is another group (Van Hove 1982, Clark and Love 1983, Dicus and Tata 1984) which feels that finite-T effects do not automatically break SUSY.

Very recently Chia (1986a, b) in a series of papers has analysed these points. He has studied several SUSY models and arrived at the conclusion that for SUSY to remain unbroken at finite T the thermal averages of all the auxiliary fields should be zero. He has based his calculation on the one-loop effective potential at finite T (Dolan and Jackiw 1974).

In this letter we wish to point out that, at finite T, vanishing of thermal averages of all the auxiliary fields alone is not sufficient for SUSY to remain unbroken. However, in addition to this there are more conditions that should simultaneously be satisfied. In this work we focus our attention on these conditions.

It is well known that the order parameter for SUSY is the energy of the ground state. If ground-state energy is zero then SUSY is unbroken, and if it is a finite non-zero positive value then SUSY is spontaneously broken. SUSY at $T\neq 0$ behaves differently from SUSY at $T=0$. Whereas, at zero temperature the vanishing of the vacuum expectation values of all the auxiliary fields in the theory guarantees that SUSY is unbroken, there is no such obvious generalisation of this critierion at non-zero temperature.

In our present calculation, taking the effective potential as the SUSY order parameter, we write down the conditions of unbroken SUSY at finite T in a quite general way. We limit our arguments up to the one-loop effective potential. To put forward our point of view we discuss two examples which were earlier discussed by Chia (1986a).

We start with a general situation in which the system consists of any number of interacting chiral multiplets represented by the superfield $S\equiv(\phi, \psi, F)$. The quantities in the parenthesis are the field contents of a single chiral multiplet. ϕ is a complex scalar field, ψ is a Majorana spinor and F is the complex scalar auxiliary field.

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The interacting theory can be best formulated by introducing a function called the superpotential $f(\phi)$. This function f depends only on scalar fields ϕ and not on their complex conjugates ϕ^*. Further, keeping renormalisability in mind $f(\phi)$ is at most cubic in the scalar fields. As it happens in SUSY theories, if necessary, auxilary fields can be eliminated by using their equations of motion

$$F^* = -\mathrm{i}\frac{\partial f(\phi)}{\partial \phi}. \tag{1}$$

The finite-T one-loop effective potential is given by the following general expression (Girardello *et al* 1981, Chia 1986a):

$$V^T = \sum_a \left|\frac{\partial f(\phi)}{\partial \phi_a}\right|^2 + \frac{1}{8}T^2 \sum_{a,b} \left|\frac{\partial^2 f(\phi)}{\partial \phi_a \partial \phi_b}\right|^2. \tag{2}$$

In equation (2) indices a and b represent the summation taken over the number of chiral multiplets considered in a given system. V^T consists of two terms. The first term represents the zero-temperature tree level contribution to the potential and it can be written as $\Sigma_a |F_a|^2$ by using equation (1). The second term in V^T gives the finite T contribution at the one-loop level. It should be noted that the $\partial^2 f(\phi)/\partial\phi_a\partial\phi_b$ factor is used in any general SUSY theory to give rise to Yukawa couplings amongst the scalar fields ϕ and the fermions ψ and also the fermion masses (Witten 1981).

Now the question is: under what circumstances is SUSY unbroken? To answer this question one finds the minima of the potential in equation (2). If there exist the values of the scalar fields ϕ at which V^T vanishes then SUSY is unbroken. If there are no values of ϕ where V^T does not vanish then V^T has positive value and SUSY is spontaneously broken. Therefore, for V^T to be zero we obtain from equation (2) that for each value of a

$$\frac{\partial f(\phi)}{\partial \phi_a} = 0 \tag{3a}$$

and

$$\frac{\partial^2 f(\phi)}{\partial \phi_a \partial \phi_b} = 0. \tag{3b}$$

This result follows from the fact that V^T has two terms and both are positive definite. So for V^T to be zero both the conditions in equations (3) must be satisfied simultaneously for some values of ϕ.

These criteria are different from that of Chia (1986a)†. He obtained only the first condition (3a) for unbroken SUSY. We would like to mention that this condition is sufficient for SUSY to be unbroken at zero T. But at finite T the second criterion (3b) should also be satisfied simultaneously.

Now we consider two SUSY models. The first model is described by the following superpotential (Chia 1986a):

$$f(\phi) = \tfrac{1}{2}m\phi_0^2 + \tfrac{1}{2}g\phi_0 \sum_{i=1}^{N} \phi_i^2 \tag{4}$$

† In Chia (1986a) equation 3(a) is connected to the vanishing of thermal averages of the auxiliary fields.

where m and g are real positive parameters. In equation (4) scalar fields ϕ_0 and ϕ_i represent an $O(N)$ singlet and N-plet respectively. The finite-T one-loop effective potential for this system is obtained easily by substituting equation (4) into equation (2). The result is

$$V^T = \left[m\phi_0 + \tfrac{1}{2}g \sum_{i=1}^{N} \phi_i^2 \right]^2 + g^2\phi_0^2 \sum_{i=1}^{N} \phi_i^2 + \tfrac{1}{8}T^2\left[m^2 + Ng^2\phi_0^2 + 2g^2 \sum_{i=1}^{N} \phi_i^2 \right]. \tag{5}$$

SUSY is unbroken if equations in (3) have simultaneous solution, i.e. if

$$\frac{\partial f}{\partial \phi_0} = m\phi_0 + \tfrac{1}{2}g \sum_i \phi_i^2 \tag{6a}$$

$$\frac{\partial f}{\partial \phi_i} = g\phi_0\phi_i \tag{6b}$$

$$\frac{\partial^2 f}{\partial \phi_0^2} = m \tag{6c}$$

$$\frac{\partial^2 f}{\partial \phi_0 \partial \phi_i} = g\phi_i \tag{6d}$$

and

$$\frac{\partial^2 f}{\partial \phi_i^2} = g\phi_0 \tag{6e}$$

vanish for some values of ϕ. One can check without difficulty that there are no values of ϕ for which all the equations in (6) vanish simultaneously. Therefore, SUSY is spontaneously broken.

This result is totally opposite to that of Chia (1986a). He has found in this model that SUSY is unbroken at finite T and that the ground state is situated at $\phi_0 = \phi_1 = \phi_2 = \ldots = \phi_N = 0$ for all T. If we evaluate the potential in equation (5) at these values of ϕ we find that $V^T = \frac{1}{8}m^2T^2$, which is positive definite, thereby indicating the spontaneous breakdown of SUSY. However, at zero T the ground state lies at $\phi_0 = \phi_1 = \phi_2 \ldots = \phi_N = 0$ for which the potential also vanishes and SUSY is unbroken.

The second example is provided by the following form of the superpotential (Chia 1986a):

$$f(\phi) = \tfrac{1}{2}m \sum_{i=1}^{N} \phi_i^2 + \tfrac{1}{2}g\phi_0 \sum_{i=1}^{N} \phi_i^2. \tag{7}$$

Again putting this $f(\phi)$ in equation (2) yields

$$V^T = (m + g\phi_0)^2 \sum_{i=1}^{N} \phi_i^2 + \tfrac{1}{4}g^2 \left| \sum_{i=1}^{N} \phi_i^2 \right|^2 + \tfrac{1}{8}T^2\left[N(m + g\phi_0)^2 + 2g^2 \sum_{i=1}^{N} \phi_i^2 \right]. \tag{8}$$

For this case equations in (3) take the following form:

$$\frac{\partial f}{\partial \phi_0}=\frac{1}{2}g\sum_{i=1}^{N}\phi_i^2 \tag{9a}$$

$$\frac{\partial f}{\partial \phi_i}=(m+g\phi_0)\phi_i \tag{9b}$$

$$\frac{\partial^2 f}{\partial \phi_0^2}=0 \tag{9c}$$

$$\frac{\partial^2 f}{\partial \phi_0 \partial \phi_i}=g\phi_i \tag{9d}$$

and

$$\frac{\partial^2 f}{\partial \phi_i^2}=(m+g\phi_0). \tag{9e}$$

An analysis of this set of equations shows that all these equations vanish simultaneously for $\phi_i=0$ $(i=1,\ldots,N)$ and $\phi_0=-m/g$. Further, evaluating the potential in equation (8) at these values shows that $V^T=0$, indicating that SUSY is unbroken. Needless to say, SUSY is unbroken in this model at $T=0$. Therefore, the superpotential of equation (7) provides an example where SUSY is unbroken at all T.

Thus we have written the correct conditions of unbroken SUSY at finite T. The examples studied in this article show that SUSY is not automatically broken at finite T.

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